Robust Multivariable Control of Large Space Structures Using Positivity

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This paper examines the robust, multivariable control of large space structures by controllers designed on a reduced-order model using positivity concepts. Controllers are designed using the DRAPER I and DRAPER II structures. Three different controller methodologies are compared: the familiar multivariable control, individual modal control, and individual sensor control. Controller robustness is measured qualitatively from the plots of the minimum singular value of the return difference matrix as a function of the frequency. All controllers, when designed to give the same total average control cost, have a very similar line-of-sight response. In addition, closed-loop stability can be maintained in the event of sensor and/or actuator failure.

Nomenclature

A,B	= state and control penalty matrices used in index of performance
CC	= control cost
F,G,H	= state representation matrices for truth model
F_a	= augmented state matrix involving truth model and estimator
F_{cl}	= design closed-loop matrix for model $(F_m - G_m K)$
F_m, G_m, H_m	= state representation matrices for reduced-order model
$G(s)$	= open-loop transfer matrix for truth model
$H_m(s)$	= open-loop transfer matrix for estimator
IP	= index of performance to find K
K	= closed-loop feedback gain matrix
K_a	= augmented matrix associated with F_a
K_F	= estimator gain matrix
ℓ	= number of actuators/sensors
m	= number of modes in truth model ($=n/2$)
n	= dimension of truth model
r	= dimension of reduced-order model
α	= constant diagonal element of state penalty matrix A
$\eta, \dot{\eta}$	= modal displacement and velocity vectors

Introduction

TODAY'S control engineers are being challenged to design control systems for large, flexible space systems. Desirable qualities in such control systems are guaranteed global asymptotic stability and the ability to remain stable over a wide envelope of disturbances and modeling errors (robustness). Much work has been done in recent years, and continues to be done, in this area.[1-5] Most of the methods proposed, however, suffer from stability problems caused by the modal truncation, which is generally essential to produce a model of sufficiently low order for use as a basis for controller design.

This paper extends work done earlier[6,7] in the application of the theory of positive real matrices to controller design for the specific case of large, flexible space structures. This theory is itself an extension of the classical study of hyperstability by Popov.[8] Using a positive real approach to controller design leads to guaranteed closed-loop system stability in the presence of erroneous modal information (frequencies and mode shapes) and unmodeled modes. While stability cannot be guaranteed in the presence of unmodeled actuator/sensor dynamics that may invalidate the positive real assumption, multivariable gain and phase margins can be used to assure a degree of robustness to these effects.

Positive Real Feedback

Before considering the controller design, it is important to understand the concept of "positivity." The following definitions and theorems are presented.[7]

Definition

A square transfer matrix $Z(s)$ is called "positive real" if 1) $Z(s)$ has real elements for real s, 2) $Z(s)$ has elements that are analytic for $Re[s]>0$, and 3) $Z(s)+Z^*(s)$ is nonnegative definite for $Re[s]>0$. For $Z(s)$ to be "strictly positive real," part 3 is replaced by 3) $Z(j\omega)+Z^*(j\omega)$ is positive definite for all real ω. A strictly positive real transfer matrix implies that the system dissipates energy.

In the case of single-input, single-output (SISO) systems, the square transfer matrices reduce to transfer functions, and the given definitions of positive realness and strict positive realness can be easily applied to check the positivity. For multiple-input, multiple-output (MIMO) systems, the transfer matrix has elements that are transfer functions, and applying the given definition is not as straightforward. For systems in state space form, we make use of the following theorem.[9]

Theorem 1

For the system described in state variable form

$$\dot{x} = Fx + Gu \tag{1}$$

$$y = Hx \tag{2}$$

the transfer matrix $G(s) = H(sI-F)^{-1}G$ is positive real if and only if there exists a symmetric positive definite matrix P and a symmetric positive semidefinite matrix Q such that

$$PF + F^T P = -Q \tag{3}$$

$$H^T = PG \tag{4}$$

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For $G(s)$ to be strictly positive real, Q must be symmetric positive definite. Implicit in Theorem 1 is the assumption that there is an equal number of inputs as outputs.

Theorem 2

The fundamental theorem of positive real feedback is given as[8] Theorem 2. Given the square transfer matrices $G(s)$ and $H(s)$ in the feedback system of Fig. 1, the system is asymptotically stable if at least one of the transfer matrices is positive real and the other transfer matrix is strictly positive real. The importance of this theorem to the control of flexible structures is as follows. The equations of motion of a large space structure can be written in terms of modal coordinates (see Meirovitch[10]). These can be put into the state space form of Eqs. (1) and (2), where the state vector is given as

$$x^T = \{\eta^T, \dot{\eta}^T\} \tag{5}$$

It is well-known that for this case, the $\ell \times \ell$ transfer matrix between y and u is positive real if y is a vector of velocity sensors colocated with a set of point actuators. This implies that the matrices G and H satisfy the relation

$$H^T = G \tag{6}$$

This result is independent of the number of modes in the model, and in fact holds for the infinite dimensional system. In this work, a positive real feedback controller is designed for a vibratory system using an observer model based on a reduced-order model of the actual structure. The reduced-order system is assumed to be in the form

$$\dot{x}_r = F_m x_r + G_m u \tag{7}$$

$$y = H_m x_r \tag{8}$$

where x_r is the r-dimensional state associated with the reduced-order model and u is the ℓ-dimensional input vector.

The dynamics of the estimator are given by

$$\dot{\hat{x}} = F_m \hat{x} + G_m u + K_F (y - H_m \hat{x}) \tag{9}$$

The control u is assumed to be a linear feedback of the estimated state

$$u = -u' = -K\hat{x} \tag{10}$$

The transfer matrix relating the observer output u' to the observer input y can be found from Eqs. (9) and (10) as

$$H_m(s) = K(sI - F_T)^{-1} K_F \tag{11}$$

with

$$F_T = (F_m - G_m K - K_F H_m) \tag{12}$$

In our studies, the feedback gain matrix K is conveniently chosen to minimize the index of performance

$$IP = \int_0^\infty (y^T A y + u^T B u)\,\mathrm{d}t \tag{13}$$

and to place the nominal poles of the reduced-order model. The matrix K_F is an $r \times \ell$ matrix of estimator gains, which is chosen to satisfy the requirements of Theorem 1 and ensure that $H_m(s)$ is strictly positive real for a given K by using[11]

$$PF_{cl} + F_{cl}^T P = -(Q - K^T H_m - H_m^T K) = -\tilde{Q} \tag{14}$$

$$F_{cl} = (F_m - G_m K) \tag{15}$$

$$K_F = P^{-1} K^T \tag{16}$$

By selecting a symmetric Q in Eq. (14) such that the matrix $\tilde{Q}$ is at least positive semidefinite, we can solve this Lyapunov equation for the unique positive definite P. The estimator gains K_F follow directly from Eq. (16).

If the full model is used as a basis for controller design, then F_m, G_m, and H_m become F, G, and H, respectively, and the eigenvalues of the full closed-loop system will simply be the eigenvalues of $(F-GK)$ and $(F-K_F H)$. Once a reduced-order model is used for controller design, this will no longer be true. This is one of the problems associated with the large space structure control problem. Because the model used must be of reduced order, "spillover" effects can in fact destabilize the weakly stable modes. The important feature of the positive real feedback approach is that the system is guaranteed to be stable regardless of spillover effects.

Spacecraft Model and Modal Reduction

Three different modal reduction techniques were used in this study of positive real controllers. These are 1) simple modal truncation (i.e., the deletion of the highest frequency modes), 2) a singular perturbation technique by Litz and Roth[12] which retains eigenvalues based on input-output characteristics, and 3) a balanced realization technique used by Gregory.[13] These modal reduction techniques were applied to the DRAPER I and DRAPER II structural models, which have been used in the past as a bench mark for space structure control design algorithms.

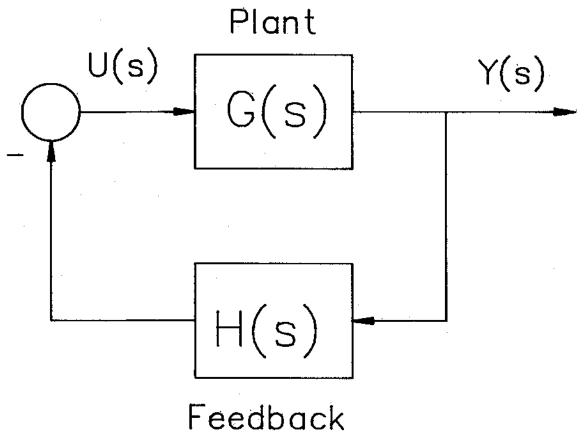

Fig. 1 General feedback concept.

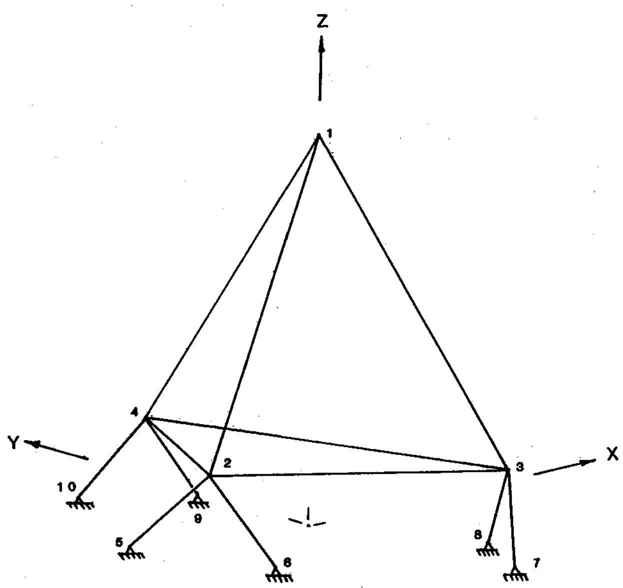

Fig. 2 DRAPER I tetrahedral truss structure.

Table 1 Modal natural frequencies for the DRAPER I model

Mode	Frequency rad/s	Frequency Hz
1	1.342	0.2136
2	1.665	0.2650
3	2.891	0.4601
4	2.957	0.4707
5	3.398	0.5408
6	4.204	0.6692
7	4.662	0.7420
8	4.755	0.7568
9	8.539	1.359
10	9.250	1.472
11	10.285	1.637
12	12.905	2.054

Table 2 Modal natural frequencies for the first 17 nonzero modes of the DRAPER II model

Mode	Frequency rad/s	Frequency Hz
1	0.7161	0.1140
2	0.9231	0.1469
3	0.9399	0.1496
4	1.1009	0.1752
5	2.8616	0.4554
6	3.5019	0.5573
7	3.7459	0.5962
8	3.8627	0.6148
9	3.9982	0.6363
10	4.0315	0.6416
11	5.1216	0.8151
12	5.1281	0.8162
13	5.1741	0.8235
14	5.7532	0.9156
15	6.1075	0.9720
16	7.2806	1.1587
17	9.7457	1.5512

The DRAPER I model[14] is a tetrahedral truss connected to the ground by three right-angled bipods as shown in Fig. 2. These bipods take on the duties of rate sensors and force actuators, all in the direction of the members, giving the colocation necessary for a positive real model. The model has 12 dynamic degrees of freedom and thus a maximum of 12 modes. The natural frequencies for the model, as found by a NASTRAN analysis, are given in Table 1.

The DRAPER II model[15] is a model of a space telescope consisting of two subsystems as shown in Fig. 3. The optical support structure contains the four optical surfaces that are assumed to be rigid and kinematically mounted onto the structure. The equipment section is assumed to be a rigid central section with two flexible solar panels cantilevered from it.

Because of the complexity of the finite-element model, a truth model for the DRAPER II model was formed using the first 50 nonzero modes, obtained from a NASTRAN analysis of the full DRAPER II model. The natural frequencies for the first 17 modes are given in Table 2. To model the small structural damping that will be present in the structure but has not been considered in the model to date, each mode had 0.2% critical damping inserted at the state space representation level. The colocated actuators and sensors are placed at the support nodes of the mirrors, which are the positions that contribute to the line-of-sight error.

Two twentieth-order (i.e. 10-mode) reduced-order models are formed from this truth model. The first is formed by applying the singular perturbation analysis to the DRAPER II structure model as in the DRAPER I case. The 10 most important modes predicted by this analysis are modes 2,4,3,9,15,6,

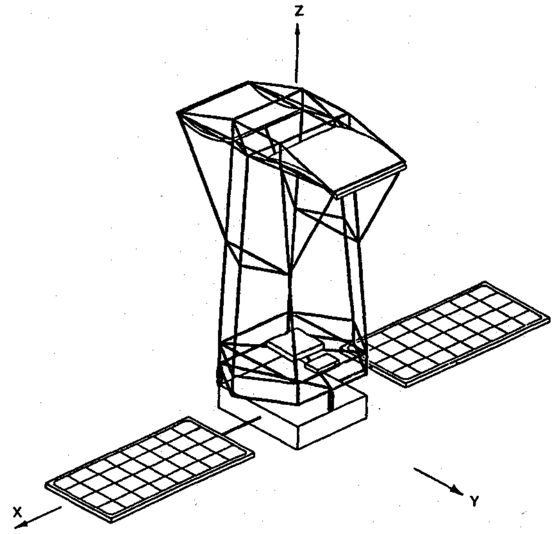

Fig. 3 DRAPER II space telescope structure.

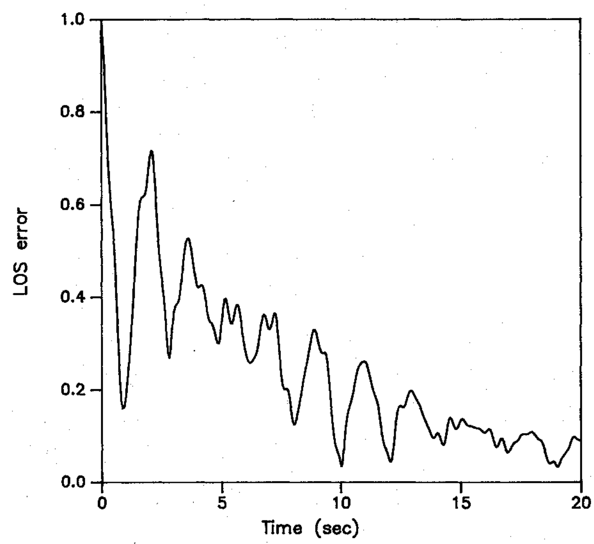

Fig. 4 LOS error for sixth-order DRAPER I multivariable controller based on HFC model.

5, 14, 7, and 16 and their rates, in that order. Because of the complex nature of this structure, simple truncation is dismissed from consideration when forming the second reduced-order model. Instead, a model is formed based on the analysis by Gregory,[13] who used the DRAPER II model as an example for his model order reduction technique. The modes kept are modes 1, 2, 4, 6, 7, 8, 9, 15, 16, and 17 and their rates.

Controller Design and Performance

Three Controller Methodologies

Three different controller methodologies were used in this study. The first will be termed the multivariable control method, although all three will in fact be controlling several actuators. In the multivariable control case, one controller is designed using the methods presented earlier such that information from all ℓ sensors is used to create control signals that are sent to all ℓ actuators. Only one K and K_F need be solved for, and the controller is an $\ell \times \ell$ MIMO controller whose order is the same as the reduced-order model.

The second control methodology is called individual mode control. In this scheme, a separate controller is designed for each of the modes kept in the reduced-order model. Each controller is therefore a second-order controller (the mode amplitude and rate), but is still MIMO since each controller receives the full output vector and produces a full control vector. The total control signal is the superposition of the control signals sent from each individual controller. The feedback function, which is the sum of r second-order positive real transfer functions, is also positive real.

The final type of controller design is called individual sensor control, whereby a separate positive real controller is designed for each colocated actuator/sensor pair. Each controller is then the same order as the reduced-order model that it is based upon, but is now reduced to a SISO configuration. The total feedback matrix is reduced to a diagonal structure in a simple type of decentralized architecture where again the feedback matrix is positive real since each component is positive real.

Each of these three types of controllers has its advantages and disadvantages. The main advantage to multivariable control is that only one control synthesis (evaluation of the feedback gain matrix K and estimator gain matrix K_F) must be done. For iteration on the gain matrices to fine-tune system performance, this could result in a significant computational savings. This form of multivariable control is mathematically the optimal way to control a MIMO system. However, this method does not allow much design flexibility in placing the poles of our controller and estimator if there are some weakly uncontrollable/unobservable modes in the reduced-order models. The individual mode controllers allow the modification of the relative gains to accommodate modes that have some uncontrollable/unobservable character. In the opposite sense to the multivariable controllers, however, more synthesis, admittedly of lower-order matrices, must be performed.

The individual sensor control scheme is a decentralized control technique that forces a diagonal feedback structure, yielding ℓ-SISO loops. These controllers will also allow the addition of extra actuator/sensor sets at any point in the control system design without necessitating a redesign of the other individual sensor controllers. This latter quality seems particularly desirable when considering, for example, the proposed Space Station, which is slated to evolve over a long period of time. A control system that can either evolve along with the spacecraft or is insensitive to changes in it is required unless a complete redesign of the control system is expected with every addition to the system.

Controller Synthesis

Actual controller synthesis involves the choice of the three matrices A, B, and Q as defined in Eqs. (13) and (14). There is considerable design flexibility in the choice of A and B to move the closed-loop eigenvalues of $(F_m - G_m K)$ and in the choice of Q to move the estimator eigenvalues of $(F_m - K_F H_m)$. In this study, A is taken as αI and B is taken as I. The same value of α is used when finding the closed-loop eigenvalues for all controller methodologies and design models. The specific values used for α in each controller design are 10^3 and 10^{10} for the DRAPER I and DRAPER II structures respectively. For the DRAPER I structure, only the x and y line-of-sight (LOS) errors are considered in the evaluation of K, whereas the x and y LOS errors and z "defocus" error (refer to Fig. 3) are considered for the DRAPER II structure.

The estimator gain matrix K_F is determined by the choice of the matrix Q in Eq. (14). In this study, Q was chosen as $Q = qI$ where the scalar q was chosen to give an equal "expected" control cost for all controllers being evaluated. The control cost was evaluated by considering the integral

$$CC = \int_0^\infty (u^T R u) = x_0^T W x_0 \tag{17}$$

where W satisfies

$$F_a^T W + W F_a = -K_a \tag{18}$$

x_0 is the initial condition on the augmented state vector, and F_a and K_a for each controller are given in the Appendix. The scalar cost was evaluated by a "pseudospectral" norm, which consists of the maximum eigenvalue of the top (leading diagonal) $n \times n$ block of W (i.e., omitting the estimator initial conditions). The specific values for the total control cost of 10^3 and 10^4 were chosen for the DRAPER I and DRAPER II structures respectively.

Actual controller synthesis involves the numerical solution of a single Riccati equation for the closed-loop feedback gain matrix K, a Lyapunov equation solution for the estimator gain matrix K_F, and a Lyapunov equation for the total control cost. These latter two equations are iterated until the final desired cost is obtained.

Controller Performance

The performance of the controllers is evaluated as a time history of an error function defined as an LOS error. For the DRAPER I model, the total error e is given as

$$e = [\mathrm{LOS}X^2 + \mathrm{LOS}Y^2]^{1/2} \tag{19}$$

where LOSX and LOSY are the x error and y error of vertex 1, respectively.

The DRAPER II model controller performance is measured with respect to a line-of-sight error algorithm given by[15]

$$\begin{bmatrix} \mathrm{LOS}X \\ \mathrm{LOS}Y \\ Z \end{bmatrix} = [\phi_1]\eta \tag{20}$$

where η is the vector of modal amplitudes, LOSX and LOSY are line-of-sight rotation errors about the x and y axes respectively, Z is the defocus term, and $[\phi_1]$ is a matrix relating modal displacements to the LOS errors.

A critical property of feedback systems is the ability to maintain performance in the face of uncertainties, i.e., they must be "robust." These uncertainties result from variations in the modeled parameters as well as plant elements that are ignored or approximated in the truth model. For example, in this study the actuator and sensor dynamics are ignored in the control design. The closed-loop system must remain stable despite the differences between the truth model and the actual plant.

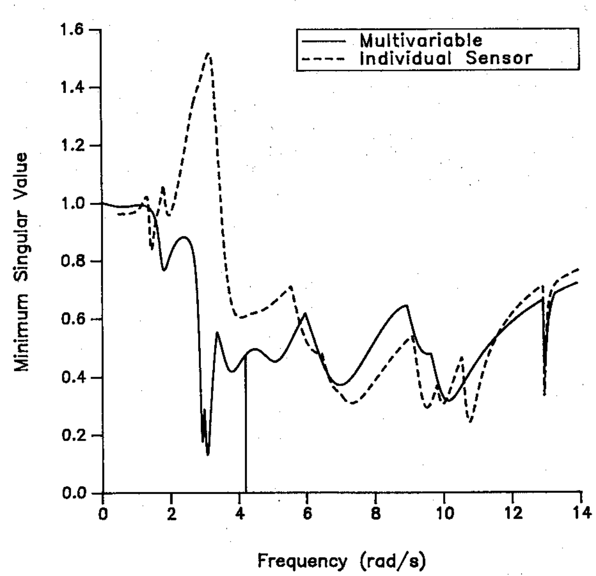

Fig. 5 $\underline{\sigma}(I+HG)$ **for DRAPER I system with sixth-order HFC based on multivariable and individual sensor controllers.**

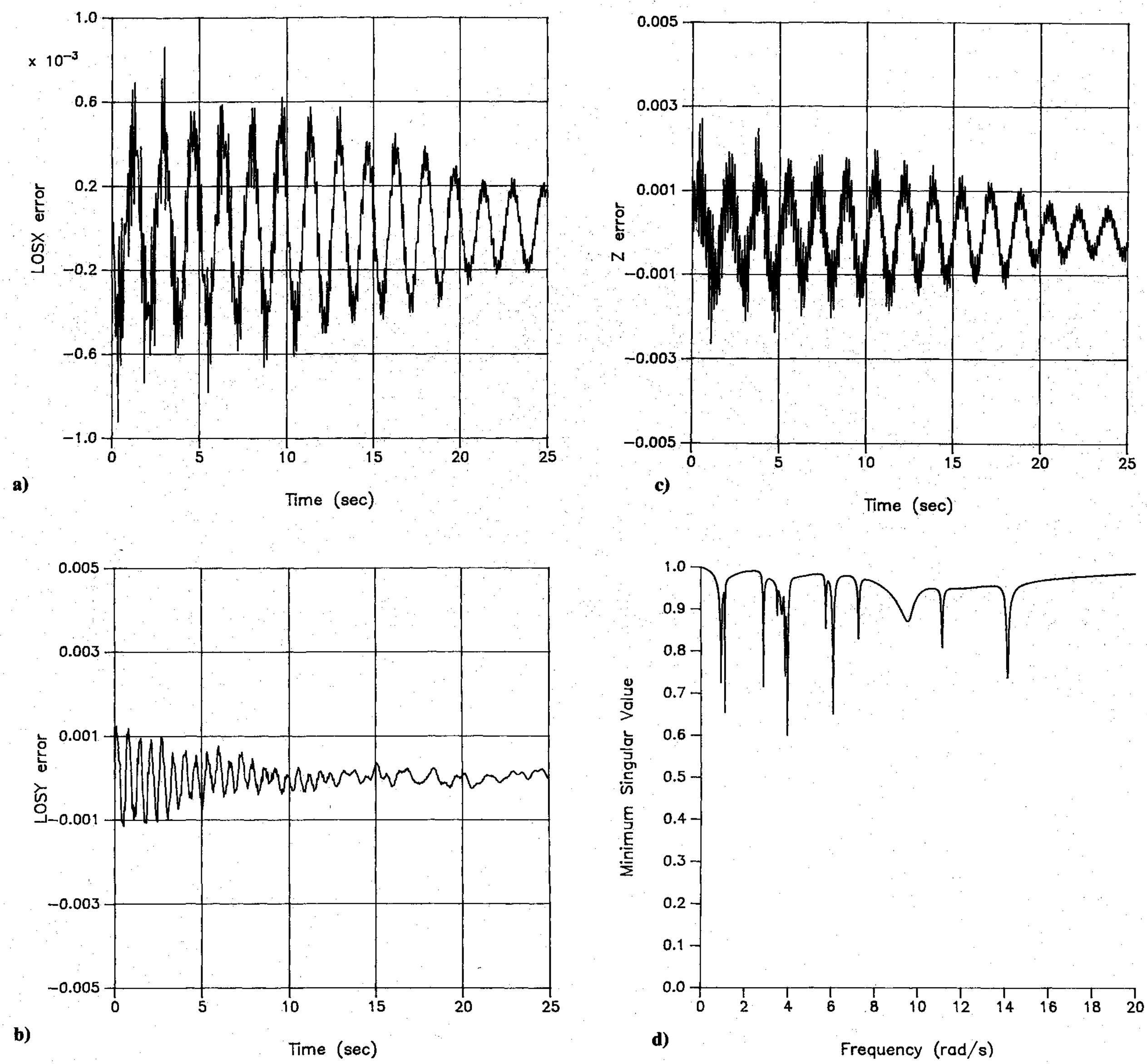

Fig. 6 Twentieth-order DRAPER II multivariable controller based on Gregory model: a) LOS x error, b) LOS y error, c) LOS z error, and d) $\underline{\sigma}(I+HG)$.

The minimum singular value of the return-difference matrix $[I+H_m(s)G(s)]$ at nominal gain is used to examine the robustness of the closed-loop plant.[16,17] Here, $G(s)=H(sI-F)^{-1}G$ and $H_m(s)$ depends on the particular form of controller used (see Appendix). The minimum singular value of a matrix D, denoted $\underline{\sigma}(D)$, gives an accurate measure of how close D is to being singular. The minimum singular value of the return-difference matrix is used to establish a sufficiency bound on the "size" of the allowable perturbations that can destabilize the system. Due to the nature of this bound, the singular value test tends to be quite conservative. Nevertheless, it gives valuable information on the relative robustness of different control designs.

Results

Throughout this discussion, controllers based on models that keep modes as predicted by the eigenvalue dominance and singular perturbation analysis will be referred to as high-frequency controllers (HFC), controllers based on the truncation of all but low-frequency modes will be called truncation controllers, and controllers based on the "balanced" approach by Gregory will be termed Gregory controllers.

Control of the DRAPER I Model

For all controller methodologies, the LOS error response was very similar, indicating that the choice of controller design technique has little effect on the error response. A typical LOS error response, in particular for a sixth-order, HFC multivariable controller acting on the full structure, is shown in Fig. 4.

Mode six, which is not a mode that is kept in any controller design model, is unobservable in the LOS error, and hence is overlooked in the calculation of feedback and estimator gains. This leads to a very poorly damped pair of poles in the closed-loop system. Although this does not affect the error response, it does affect the robustness as seen through the minimum singular value plots.

Figure 5 shows the minimum singular value plot for the full system with sixth-order HFC multivariable and individual sensor controllers. In the multivariable controller case, the narrow downward spike at the frequency corresponding to mode six is indicative of a poorly damped mode at this frequency, and indicates that the system exhibits virtually no robustness at this frequency. The return-difference matrix is nearly singular at this frequency, and only a very small input perturbation is required to make the system unstable.

Adding more modes to the reduced-order model, even mode six itself, does not increase the damping of mode six for these multivariable controllers. In fact, adding additional modes that are observable in the LOS error but only weakly observable in the output causes a degradation in the error response. It is not possible to increase the damping of mode six by selectively changing the feedback gain matrix K, due to the optimization criterion considered here. Increasing the feedback gains across the board will help, but at the expense of more control effort.

With the use of an individual modal controller, such weakly observable modes can be addressed directly by iteration on the weights used to give acceptable pole placement. However, this will only be true for modes that are kept in the controller design model. Incorporating mode six into the controller does remove the singularity at that frequency but has no observable effect on the time response.

There was a dramatic improvement in the robustness of the closed-loop system when individual sensor control was used, as indicated by the minimum singular value plot for this case. There is no easy explanation for this phenomenon. The individually designed controllers, producing a diagonal feedback matrix $H(s)$, apparently have a synergistic effect in improving the damping in all modes, thus producing the improved robustness. Further studies to quantitatively define this improvement are an area of future research.

Control of the DRAPER II Model

For the DRAPER II model, using the twentieth-order reduced model, controllers were designed using the multivariable synthesis approach and the individual modal control synthesis. Even though the controller synthesis is straightforward, an individual sensor control scheme was not analyzed, since the numerical verification and simulation of this design would have been prohibitive at this time (23 nodal control points and the 50-mode truth model lead to a total system order of 560).

Figures 6a-6c show the LOS error responses for the multivariable control using the Gregory model. These are obtained using, as an initial condition, a unit velocity in the y direction of the top four nodes in the finite-element model. Figure 6d gives the minimum singular value plot for this same case. These results are comparable to, although slightly better than, the controller designed using the singular perturbation approach by Litz, indicating that for this case, the internally balanced representation of Gregory gives a better approximation to model reduction.[18]

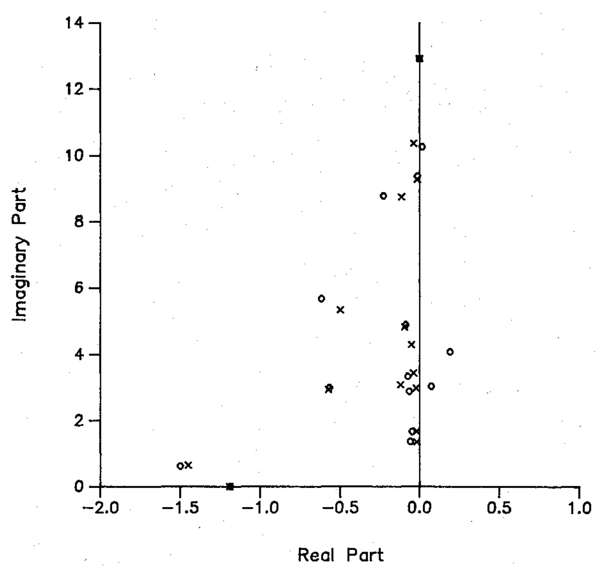

Fig. 7 Sixth-order DRAPER I individual mode controller based on HFC model: o = failure in actuators 1, 4 and sensor 2, 6, x = failure in actuator/sensor pairs 1, 2, 4, 6.

In all cases, the closed-loop systems exhibit asymptotic stability. The minimum singular value plots require a great deal of computational effort, and for even more complicated structures with larger truth models, these requirements may become excessive. There will therefore be a limit to the practicality of finding these plots as the structures in question become more complicated than this relatively "simple" space telescope.

System Performance in the Presence of Actuator/Sensor Failures

For autonomous systems in space, failures in actuators and/or sensors can cause instabilities in the control scheme, which could be fatal to the structure unless the controller is turned off. Easy access for repair is not generally possible, so the system would be considered uncontrolled and lost, at least for a significant length of time. It is desirable that the control scheme be robust with respect to actuator and/or sensor failure. Note that since the individual sensor controller results in a diagonal $H_m(s)$, it will necessarily retain positive realness, hence stability, in the presence of actuator/sensor failures.

It is assumed that if any sensor and/or actuator malfunctions and fails, then the device fails "off." Therefore, if actuator i fails, effectively the ith row of K becomes all zeros. If sensor j fails, effectively the jth column of K_F becomes all zeros. The feedback transfer matrix, which is in general

$$H_m(s) = K(sI - F_T)^{-1} K_F$$

as in Eq. (11), will therefore have row i all zeros for a failure in the ith actuator and will have column j all zeros for a failure in the jth sensor. For any general i and j, these failures may destroy the strict positive realness requirement of $H_m(s)$ from Theorem 2. The system will then not be guaranteed stable by the positive real theorem, although it still may in fact be stable, depending on the particular system and actuators/sensors involved. This is verified in numerical simulations: certain combinations of actuators and sensors can be failed and still give a stable system, but there are numerous combinations of actuator/sensor failures for which the system is unstable.

Using the positivity approach, we can, however, guarantee closed-loop stability even in the presence of multiple failures. The rationale for this statement is seen in the following theorem.

Theorem 3

Any multivariable positive real feedback matrix $H(s)$ will remain positive real if any row and corresponding column is set equal to zero. The proof of this theorem is straightforward, and follows directly from the definition of positive realness.

Theorem 3 suggests a method by which the controlled system can be forced to remain at least marginally stable in the presence of actuator/sensor failures. Whenever actuator (sensor) i fails, if then the corresponding sensor (actuator) is turned off immediately, the feedback transfer matrix will in effect have row and column i all zeros. The control system will still be guaranteed to be at least marginally stable by Theorem 3. Of course, system performance may be compromised because the control effort is less, but at least the control system will remain stable.

To illustrate the preceding discussion, consider the DRAPER I structure, which has six actuator/sensor pairs. Simulations are performed for the full augmented DRAPER I system, using all three controller methodologies based on the sixth-order design models, assuming failures in actuators 2 and 6 and in sensors 1 and 4. Obviously, this would be considered a catastrophic failure mode, and in every case (except individual sensor control) the controlled system is unstable in the presence of these failures. If the corresponding sensors and actuators are now turned off also (essentially "failure" in actuators 1 and 4 and in sensors 2 and 6), the controlled system is now stable.

A closed-loop pole plot for both failure modes is given in Fig. 7 for the specific case of a sixth-order HFC individual mode controller acting on the full structure, clearly showing how the system will become stable once both colocated actuators and sensors have "failed." These results are typical of those that occurred for both the multivariable and individual controllers of any order based on any model reduction technique.

Conclusions

Large space structures can be controlled effectively with guaranteed closed-loop stability using positivity concepts. For the DRAPER I structure, all controller methodologies studied provide a similar LOS error response if designed to achieve some constant total "expected" control cost.

The individual sensor controllers exhibit by far the best system robustness of any controller methodology. The main improvement is in the low-frequency area within the controller bandwidth. High-frequency robustness is similar for all controllers.

For the DRAPER II structure, preliminary results indicate that the model reduction technique by Gregory produces a more faithful representation of the full system than does the singular perturbation method. The multivariable control methodology gives better overall closed-loop characteristics than the individual modal controllers. These results will need to be extended in future work.

The control of large space structures using positivity concepts is robust with respect to actuator/sensor failures. Marginal stability, at least, is guaranteed if the corresponding sensor/actuator is turned off for any given actuator/sensor failure.

Appendix: Closed-Loop Eigenvalues and Full System Response

Full Multivariable Control

Estimator dynamics are

$$\dot{\hat{x}} = F_m\hat{x} + G_m u + K_F(y-\hat{y}) \tag{A1}$$

$$\hat{y} = H_m\hat{x} \tag{A2}$$

$$u = -K\hat{x} \tag{A3}$$

Form the augmented state

$$\tilde{x}^T = \{x^T\ \hat{x}^T\} \tag{A4}$$

Then

$$\dot{\tilde{x}} = F_a\tilde{x} \tag{A5}$$

where

$$F_a = \left[\begin{array}{c|c} F & -GK \\ \hline K_F H & F_T \end{array}\right] \tag{A6}$$

$$F_T = F_m - G_m K - K_F H_m \tag{A7}$$

With this as F_a, the K_a in Eq. (18) becomes

$$K_a = \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & K^T BK \end{array}\right] \tag{A8}$$

and the feedback transfer matrix is

$$H_m(s) = K(sI-F_T)^{-1}K_F \tag{A9}$$

Individual Modal Control

The estimator models are given by

$$\dot{\hat{x}}_i = F_{mi}\hat{x}_i + G_{mi}u_i + K_{Fi}(y-\hat{y}_i) \tag{A10}$$

$$\hat{y}_i = H_{mi}\hat{x}_i \tag{A11}$$

$$u_i = -K_i\hat{x}_i \tag{A12}$$

where $i = 1,\ldots,r/2$. The state for each modal controller is

$$\hat{x}_i^T = \{\hat{x}_i\ \dot{\hat{x}}_i\} \tag{A13}$$

Again, form an estimator state as

$$\hat{x}^T = \{\hat{x}_1\ \hat{x}_2 \ldots \dot{\hat{x}}_1\ \dot{\hat{x}}_2 \ldots\} \tag{A14}$$

$$K = \{K_{11}\ K_{21} \ldots K_{12}\ K_{22} \ldots\} \tag{A15}$$

$$K_F^T = [K_{F_{11}}^T\ K_{F_{21}}^T \ldots K_{F_{12}}^T\ K_{F_{22}}^T \ldots] \tag{A16}$$

where K_{ij} is the jth column of K_i and K_{Fij} is the jth row of K_{Fi}. If K and K_F are defined in this manner, F_a, K_a, and $H_m(s)$ will be the same in this case as in the case of full multivariable control.

Individual Sensor Control

Estimator dynamics are

$$\dot{\hat{x}}_i = F_{mi}\hat{x}_i + g_{mi}u_i + K_{Fi}(y_i-\hat{y}_i) \tag{A17}$$

$$\hat{y}_i = h_{mi}\hat{x}_i \tag{A18}$$

$$u_i = -K_i\hat{x}_i \tag{A19}$$

for $i = 1,\ldots,\ell$. Now, form the augmented state

$$\tilde{x}^T = \{x^T\ \hat{x}_1^T \ldots \hat{x}_\ell^T\} \tag{A20}$$

and then Eq. (A5) will hold with

$$F_a = \left[\begin{array}{c|c|c|c} F & -g_1K_1 & \ldots & -g_\ell K_\ell \\ \hline K_{F_1}h_1 & (F_T)_1 & \ldots & 0 \\ \hline \vdots & \vdots & & \vdots \\ \hline K_{F\ell}h_\ell & 0 & \ldots & (F_T)_\ell \end{array}\right] \tag{A21}$$

$$K_a = \left[\begin{array}{c|c|c|c} 0 & 0 & \ldots & 0 \\ \hline 0 & K_1^T RK_1 & \ldots & 0 \\ \hline \vdots & \vdots & & \vdots \\ \hline 0 & 0 & \ldots & K_\ell^T RK_\ell \end{array}\right] \tag{A22}$$

$$(F_T)_i = F_{mi} - g_{mi}K_i - K_{Fi}h_{mi} \tag{A23}$$

Here g_{mi} is the ith column of G_m, h_{mi} is the ith row of H_m, g_i is the ith row of G, and h_i is the ith column of H. In this case, the feedback transfer matrix will be diagonal, with

$$[H_m(s)]_{ii} = K_i(sI-F_{Ti})^{-1}K_{Fi} \tag{A24}$$

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